129A HW # 7 Solutions

1. Neutrinos have all helicity $-1/2$ while anti-neutrinos $+1/2$. The parity flips all helicities, and hence takes a neutrino with helicity $-1/2$ to a neutrino with helicity $+1/2$; such a state doesn't exist in Nature! But if you further do charge conjugation, it is now transformed to an anti-neutrino with helicity $+1/2$, which does exist. Therefore, doing C and P both at the same time brings an existent state to another existent state. CP has at least a good chance of being a symmetry of even the weak interaction.

2. $(CP)|K_1\rangle = CP(|K^0\rangle - |\overline{K^0}\rangle)/\sqrt{2} = -C(|K^0\rangle - |\overline{K^0}\rangle)/\sqrt{2} = -(|\overline{K^0}\rangle - |K^0\rangle)/\sqrt{2} = |K_1\rangle$, $(CP)|K_2\rangle = CP(|K^0\rangle + |\overline{K^0}\rangle)/\sqrt{2} = -C(|K^0\rangle + |\overline{K^0}\rangle)/\sqrt{2} = -(|\overline{K^0}\rangle + |K^0\rangle)/\sqrt{2} = -|K_2\rangle$. Therefore, they are eigenstates of the CP operator and have eigenvalues $+1$, -1, respectively.

3. Since all pions are in S-wave, we do not need to consider the $(-1)^L$ sign under parity for $L \neq 0$ spherical harmonics. Under parity, therefore, all pions have eigenvalues -1, which is the intrinsic parity of pions. Under charge conjugation, π^0 have eigenvalue $+1$ (remember it decays into $\gamma\gamma$). Then the eigenvalue under CP is simply determined by the number of pions, and hence $|\pi^0\pi^0\rangle$ state has $CP = +1$ and $|\pi^0\pi^0\pi^0\rangle$ state $CP = -1$. Assuming conservation of CP, $K_1 \to \pi^0\pi^0$ and $K_2 \to \pi^0\pi^0\pi^0$, but other process such as $K_2 \to \pi^0\pi^0$ is forbidden.

4. K_1 which decays into two π^0 is much shorter lived than K_2 which decays into three π^0 because a kaon has barely enough mass to produce three π^0 and hence such process occurs slowly. This is called the phase space suppression. In the booklet, the neutral kaon state which decays into $\pi^0\pi^0$ is denoted as K_S^0, and has a lifetime of $\tau_S = (0.8927 \pm 0.0009) \times 10^{-10}$ sec. K_S^0 stands for Short-lived kaon. On the other hand, the one decays into $\pi^0\pi^0\pi^0$ is called K_L^0, Long-lived kaon, and has a lifetime of $\tau_L = (5.17 \pm 0.04) \times 10^{-8}$ sec; almost three orders of magnitudes longer lived! We are tempted to identify K_S with K_1 and K_L with K_2.

5. Strong interaction preserves strangeness. In the process $pn \to \Lambda p +$ neutral K-meson, the initial state has vanishing strangeness, while Λ baryon has strangeness -1 (p.96 of booklet). Therefore, the neutral K-meson must have strangeness $+1$ to conserve strangeness, and hence must be K^0. (Recall that the strange quark s carries strangeness -1 due to historic reasons. Blame Nishijima and Gell-Mann!)

6. The created neutral K^0 is a 50-50 mixture of K_L and K_S: $|K^0\rangle = (|K_L\rangle + |K_S\rangle)/\sqrt{2}$. For K_S fraction to be less than 10^{-5}, we need time t in the rest frame

$$\frac{0.5e^{-t/\tau_S}}{0.5e^{-t/\tau_S} + 0.5e^{-t/\tau_L}} < 10^{-5},$$

or, $t > 1.03 \times 10^{-9}$ sec. With an energy $E = 10$ GeV, the kaons live longer because of the time dilation effect, and we need to wait for $\gamma t = (E/m_K c^2)t = 2.07 \times 10^{-8}$ sec. Within this time interval, they go over a distance $(\gamma t)c\beta = 6.20$ m. You need a beam line longer than this to make sure that the K_S fraction is less than 10^{-5}.

7. From the contamination of K_S in the kaon beam, we expect at most $10^{-5} \times \mathrm{BR}(K_S \to \pi^0\pi^0) = 3.14 \times 10^{-6}$ of $\pi^0\pi^0$ final states among all kaons. On the other hand, we see about 10^{-3} of kaons decaying into $\pi^0\pi^0$ (more accurately, $(9.36 \pm 0.40) \times 10^{-4}$ according to the booklet). This either means: (1) K_L state contains a little bit of K_1 which decays into $\pi^0\pi^0$ and hence the mass eigenstate does not respect CP, or (2) K_L state is still a pure K_2 state with $CP = -1$, but there is a CP-violating decay $K_2 \to \pi^0\pi^0$. In either case, CP is not respected in the neutral kaon system. (In the HW #8, we will see that the first interpretation gives the accurate description of both $K_L \to \pi^0\pi^0$ and $\pi^+\pi^-$.)

N.B. In practice, $\pi^+\pi^-$ is much easier to look for than $\pi^0\pi^0$ and indeed the former was the process found first by Cronin, Fitzsch and collaborators. I chose $\pi^0\pi^0$ for this homework because the analysis of CP eigenvalue is slightly easier than $\pi^+\pi^-$. You are welcome to prove that $\pi^+\pi^-$ with $L = 0$ has $CP = +1$, which is not difficult.

N.B. Since the longer-lived kaon has a slight mixture of K_1, we don't use K_2 to refer to the mass eigenstate, but rather K_L. The mass eigenstate K_S is also not a pure K_1, but has a small mixture of K_2.