129A Midterm (due Oct 31)

1. On February 23, 1987, a supernova (named SN1987A) exploded in Large Magellanic Cloud, which is about 50 kpc away from the Earth. Kamiokande Collaboration and IMB Collaboration detected a burst of neutrinos and reported them in K. Hirata *et al.*, *Phys. Rev. Lett.* **58**, 1490 (1987); R.M. Bionta *et al*, *Phys. Rev. Lett.* **58**, 1494 (1987). Answer the following questions.

(a) Calculate the energy of neutrinos using the table of events in Hirata paper, from the observed electron (actually believed to positron) energy and the angle from the SN1987A direction for each of them. Assume that all events are caused by the reaction $\bar{\nu}_e p \to e^+ n$, and neglect the error bars.

(b) Obtain the upper bound on the neutrino mass. Since the energy of neutrinos varied from one event to another, a massive neutrino would have different velocities and hence different arrival times. From the observed spread in the arrival times, one can place an upper bound on the neutrino mass. (The last three events arrived more than 9 seconds later from the first event, and there is a dispute if they came from SN1987A. Discard them and use only the first 9 events.)

2. The following processes have not been seen. This is understood as consequences of certain conservation laws. Explain what conservation law forbids each process. (a) $p \to e^+\pi^0$, (b) $\mu^- \to e^-e^-e^+$, (c) $n \to p\nu_e\bar{\nu}_e$, (d) $\tau^- \to \mu\gamma$, (e) $n \to p\mu^-\bar{\nu}_\mu$, (f) $K^0 \to \mu^+e^-$, (g) $\mu^- \to \pi^-\nu_\mu$.

3. The isospin is a good quantum number in strong interaction. There are spin 1 mesons (ρ^-, ρ^0, ρ^+) which form isospin 1 multiplet and another spin 1 meson ω which has isospin 0. They are understood as the bound states $(d\bar{u}, (u\bar{u} - d\bar{d})/\sqrt{2}, u\bar{d})$, and $(u\bar{u} + d\bar{d})/\sqrt{2}$, respectively.

(a) Even though both ρ and ω share the same constituents, they decay very differently. Look up the Booklet and find the most dominant decay modes for them.

(b) Explain why $\rho^0 \to \pi^+\pi^-$ is allowed but $\rho^0 \to \pi^0\pi^0$ is not.

(c) Construct wavefunctions for two-pion states both for total isospin $I = 1$ and 0, and determine the allowed values for the relative orbital angular momentum L.

(d) Explain why ω cannot decay into two pions as long as isospin is conserved.

(e) Look up the Booklet and find that $\omega \to \pi^+\pi^-$ does occur but is quite suppressed. What interaction causes this decay?

4. We would like to construct wavefunctions of baryons using quarks. The wavefunctions depend on flavor, spin, and color of quarks. Assume the color part of the wavefunction is "white," *i.e.*, $(RGB + GBR + BRG - RBG - BGR - GRB)/\sqrt{6}$ and is hence totally anti-symmetric. Because of Fermi statistics of quarks, the flavor and spin part of the wavefunction must be then totally symmetric. Assume that the orbital wave function is all S-wave and you do not need to take it into consideration. Answer the following questions.

(a) There are six possibilities for flavor and spin: $u^\uparrow$, $u^\downarrow$, $d^\uparrow$, $d^\downarrow$, $s^\uparrow$, $s^\downarrow$. How many possible states are there for three quarks which are completely symmetric?

(b) How many states are there for baryon octet and baryon decouplet including spin multiplities? How does the total number compare to the counting in the previous question?

(c) The Δ^{++} baryon is given by the wavefunction

$$|\Delta^{++}, 3/2\rangle = |u^\uparrow u^\uparrow u^\uparrow\rangle,$$

where $m = 3/2$ refers to the z-component of the spin. Note that it is totally symmetric under the exchange of any pair of the quarks. Construct the wavefunction for $|\Delta^{++}, 1/2\rangle$ state by acting lowering operator for spin on Δ^{++} state. The following formulæ are useful:

$$\begin{aligned} J^+|j,m\rangle &= \sqrt{j(j+1) - m(m+1)}\,|j,m+1\rangle, \\ J^-|j,m\rangle &= \sqrt{j(j+1) - m(m-1)}\,|j,m-1\rangle. \end{aligned}$$

When $J^\pm$ acts on a multi-particle state, it is given by $J^\pm = J_1^\pm + J_2^\pm + J_3^\pm$ where each of $J_i^\pm$ acts only on the ith particle.

(d) Construct the wavefunction for $|\Delta^+, 1/2\rangle$ by acting I^- on $|\Delta^{++}, 1/2\rangle$ state. The isospin operators satisfy the exactly the same formulæ as those for spin operators above.

(e) Show that the following wavefunction for the proton

$$|p,1/2\rangle = \frac{1}{3\sqrt{2}}\Big(2|u^\uparrow u^\uparrow d^\downarrow\rangle + 2|u^\uparrow d^\downarrow u^\uparrow\rangle + 2|d^\downarrow u^\uparrow u^\uparrow\rangle$$
$$-|u^\uparrow u^\downarrow d^\uparrow\rangle - |u^\downarrow u^\uparrow d^\uparrow\rangle - |d^\uparrow u^\downarrow u^\uparrow\rangle - |d^\uparrow u^\uparrow u^\downarrow\rangle - |u^\uparrow d^\uparrow u^\downarrow\rangle - |u^\downarrow d^\uparrow u^\uparrow\rangle\Big)$$

is orthogonal to $|\Delta^+, 1/2\rangle$ state, and also that it is totally symmetric.

(f) Check that the above proton wavefunction has isospin 1/2 by showing that $I^3|p,1/2\rangle = 1/2$ and $I^+|p,1/2\rangle = 0$.

(g) Check that the above proton wavefunction has spin 1/2 by showing that $J^3|p,1/2\rangle = 1/2$ and $J^+|p,1/2\rangle = 0$.

(h) Calculate the expectation value of the magnetic moment operator with the above proton wavefunction. Assume each of the quarks has a mass of about $m_u \approx m_d \approx m_p/3$, and also has $g = 2$ as predicted by the Dirac equation. Compare the value to the magnetic moment of proton given in the Booklet.