129B HW # 5 (due Feb 27)

We would like to check the consistency of the $SU(2)_L \times U(1)_Y$ gauge theory by "measuring" $\sin^2\theta_W$ in many different ways. Calculate $\sin^2\theta_W$ from the observables given in the booklet using the following formulæincluding errors. Use $\alpha(m_Z) = 1/128.9$.

1. $m_Z = \frac{1}{2}\frac{e}{\sin\theta_W \cos\theta_W} v$, where v is given by the relation $v^2 = G_F^{-1}/\sqrt{2}$.

2. $m_W = \frac{1}{2}\frac{e}{\sin\theta_W} v$.

3. $A_f = \frac{((g_{f_L})^2 - (g_{f_R})^2)}{((g_{f_L})^2 + (g_{f_R})^2)}$, where $g_f = I_3^f - Q_f \sin^2\theta_W$. Use measured values of A_e, A_τ, A_b, A_c (see **optional** for their physical meaning).

4. The partial widths $\Gamma(Z \to f\bar{f}) = N_c G_F m_Z^3 (g_{f_L}^2 + g_{f_R}^2)/3\sqrt{2}\pi$. Use the measured values of $\Gamma(Z \to l^+l^-)$, $\Gamma(Z \to \bar{\nu}\nu)$ (invisible).

optional We would like to calculate the forward-backward asymmetry A_{FB}^f in $e^+e^- \to Z \to f\bar{f}$ (f can be muon, tau, bottom, charm, etc). It is defined by

$$A_{FB}^f = \frac{\#\text{events}(\cos\theta_f > 0) - \#\text{events}(\cos\theta_f < 0)}{\#\text{events}(\cos\theta_f > 0) + \#\text{events}(\cos\theta_f < 0)}. \tag{1}$$

The polar angle is measured from the direction of the electron beam.

1. From the angular momentum conservation, argue that the process $e_L^- e_R^+ \to f_L \bar{f}_R$ has the angular distribution of $(1 + \cos\theta_f)^2$.

2. What are the angular distributions of the processes $e_L^- e_R^+ \to f_R \bar{f}_L$, $e_R^- e_L^+ \to f_L \bar{f}_R$, $e_R^- e_L^+ \to f_R \bar{f}_L$?

3. Show that 7/8 of the events in the process $e_L^- e_R^+ \to f_L \bar{f}_R$ appear in "forward" hemisphere: $\cos\theta_f > 0$, and only 1/8 of them appear in "backward" hemisphere $\cos\theta_f < 0$.

4. Recall that the cross section of the process $e_L^- e_R^+ \to f_L \bar{f}_R$ is proportional to the coupling factors $(g_{e_L})^2 (g_{f_L})^2$, because the Z boson couples to e_L^- when it is created and then to f_L when it decays. 7/8 of them appear in the forward hemisphere and 1/8 in the backward hemisphere. Show that the total number of forward events (summed over helicities) is proportional to

$$\frac{7}{8}(g_{e_L})^2(g_{f_L})^2 + \frac{1}{8}(g_{e_L})^2(g_{f_R})^2 + \frac{1}{8}(g_{e_R})^2(g_{f_L})^2 + \frac{7}{8}(g_{e_R})^2(g_{f_R})^2, \tag{2}$$

and the number of backward events to the same expression with 7/8 and 1/8 interchanged. (Assume that the electron beam isn't polarized)

5. Show that $A_{FB}^f = \frac{3}{4} A_e A_f$. (This is how A_l, A_c, A_b are measured!)